The Sigma Notation $\sum$

The Greek letter $\sum$ (sigma) represent the sum.

The sigma notation is used to measure central tendency.

❖ Sigma Notation : Introduction to Summation

Sigma Notation ($\sum$)

Example1.

$$\sum_{i=1}^{5} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = 55$$

i=1 on bottom tell us to start with i=1

5 on top tells us to finish with i=5

For each number i that we count

i=1 $i^2 = 1^2$ =1

i=2 $i^2 = 2^2$ =4

i=3 $i^2 = 3^2$ =9

i=4 $i^2 = 4^2$ =16

i=5 $i^2 = 5^2$ =25

then the summation tells us the sum of the results.

Example2. $$\sum_{j=1}^{3} (2j + 5) = (2(1) + 5) + (2(2) + 5) + (2(3) + 5) = 27$$

j=1 : 2j+5 2(1)+5 =7

j=2 : 2j+5 2(2)+5 =9

j=3 : 2j+5 2(3)+5 =11

$\sum$ tells us the sum of results.

i.e 7+9+11 =27

Example3.

$$\sum_{r=1}^{3} \frac{r}{2} \quad = \frac{1}{2} + \frac{2}{2} + \frac{3}{2} = \frac{6}{2} = 3$$

We can write it as

$$\sum_{j=1}^{3} \frac{j}{2}$$

(here j and r are dummy indices)

- ➔ We can use i,j,k,r,m,n… etc for indices.

❖ Simplification Rules: - Sigma Notation

A) Summation of constants.

Example

$$\sum_{k=1}^{5} 3 = 3 + 3 + 3 + 3 + 3 = 3 * 5$$

=15

$$\sum_{r=1}^{6} 8 = \quad 8 * 6 \quad = 48$$

We summing constants we can multiply the constant by the number of indices we count.

B) Commutative property of sigma notation.

Example

$$\sum_{i=1}^{3} i^3 + 3i = (1^3 + 3(1)) + \left(2^3 + 3(2)\right) + (3^3 + 3(3))$$

$=1^3 + 2^3 + 3^3$+(3(1)+3(2)+3(3))

$$= \quad \sum_{i=1}^{3} i^3 + \sum_{i=1}^{3} 3i$$

This is due to the commutative property

a+b = b+a

We can add the term in any order.

C) Distributive property of sigma notation

Example

$$= \sum_{i=1}^{3} i^2 = 14$$

$$= \sum_{i=1}^{3} 4i^2 = 4(1)^2 + 4(2)^2 + 4(3)^2 = 4\{1^2 + 2^2 + 3^2\}$$

$= 4\ (\sum_{i=1}^{3} i^2)$

Thus we can write,

$\sum_{i=1}^{5} 5(i)^3$ As $5\sum_{i=1}^{5} (i)^3$

This is due to **distributive property**.

i.e **a (b+c) = ab+ac**

In other words, constants inside the summed expression can be pulled outside.