Laws of Exponents for Real Numbers

If a, b are positive real numbers and m, n a rational numbers then the following results hold :

I. $a^m \cdot a^n = a^{m+n}$

II. $(a^m)^n = a^{mn}$

III. $\dfrac{a^m}{a^n} = a^{m-n}$

IV. $a^m \cdot b^m = (ab)^m$

V. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

VI. $a^{-n} = \left(\dfrac{1}{a}\right)^n = \dfrac{1}{a^n}$

VII. $a^n = b^n \qquad n \neq 0 \qquad \Rightarrow a = b$

VIII. $a^m = a^n \qquad n \neq 0 \qquad \Rightarrow m = n \qquad$ provided $a \neq 1$

Examples :

I. $2^3 \cdot 2^4 = 2^{3+4} = 2^7$

II. $((3)^2)^2 = 3^{2 \times 2} = 3^4 = 81$

III. $\dfrac{3^6}{3^3} = (3)^{6-3} = 3^3 = 27$

IV. $3^3 \times 5^3 = (3 \times 5)^3 = (3)^2 = 3375$

V. $\left(\dfrac{2}{3}\right)^3 = \dfrac{2^3}{3^3} = \dfrac{8}{27}$

VI. $2^{-7} = \left(\dfrac{1}{2}\right)^7 = \dfrac{1}{2^7} = \dfrac{1}{128}$

Solve

$(-4)^{-3} \times (5)^{-3} \times (-5)^{-3} = (-4 \times 5 \times -5)^{-3}$

$$= (100)^{-3}$$

$$= \dfrac{1}{100^3}$$