Basic Probability Definitions:
Probability Definitions and Notation

Introduction

Definition

probability —the degree of belief in the truth orfalsity of a statement

Range of uncertainty from 0 to 1

Certain statement is true: probability 1 Certain statement is false: probability 0

Example Statement x: "It is raining."

1 Notation

$P(x)$ probability of x

$\sim x$ negation of statement x

Law of excluded middle

$$P(x) + P(\sim x) = 1$$

Probability of a statement and the probability of the negation of a statement must sum to 1.

If $P(x) = 1$, then $P(\sim x) = 0$, and vice versa.

In general, all outcomes of a probability distribution must sum to 1.

Definitions

probability distribution—collection of statements that are *exclusive* and *exhaustive*

exclusive—given complete information, no more than one of the statements can be true

exhaustive—given complete information, at least one of the statements must be true A

distribution X consisting of n statements would be denoted

$$X = \{x_1, x_2, x_3, \ldots, x_n\}.$$

The probability of each statement must sum to 1, which is denoted.

$$P(x_1) + P(x_2) + P(x_3) + \ldots + P(x_n) = 1.$$

2 Principle of indifference

For the i-th outcome x_i in a distribution with n possible outcomes, $P(x_1) = \frac{1}{n}$

Example: Drawing an ace of spades from a well-shuffled deck of 52 cards. The probability of drawing the ace of spades is $\frac{1}{52}$

General statement

When there is no basis to choose some outcomes as more likely than others,

$$P(\text{event}) = \frac{\text{number of outcomes as defined in event}}{\text{total number of possible outcomes in universe}}.$$

Example: Event is drawing a queen, which has four outcomes in the event. The total number of outcomes is 52, so the probability of drawing a queen is $\frac{4}{52} = \frac{1}{13}$.

Example: Event is rolling an even number on a six-sided die, which has three outcomes in the event. The total number of outcomes is 6, so the probability of rolling an even is $\frac{3}{6} = \frac{1}{2}$.